
MAGNET STRENGTH MEASUREMENT FROM BPM DATA

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Outline

- Physical introduction
- Limits of the model
- Code description
- Experimental issues
- SIS-18 benchmarks
- RHIC benchmarks

Physical introduction

Is it possible to derive the potential V observing the trajectory of a particle?

Dealing with a particle beam moving inside an accelerator

- the magnet potential is known:

$$V(s) = -\Re \left[\sum_{n \geq 1} (K_n(s) + iJ_n(s)) \frac{(x + iy)^{n+1}}{(n+1)!} \right]$$

we need to derive only the coefficients K_n, J_n ;

- the betatron motion is (quasi)periodic \Rightarrow Fourier analysis and Normal Form can be used.

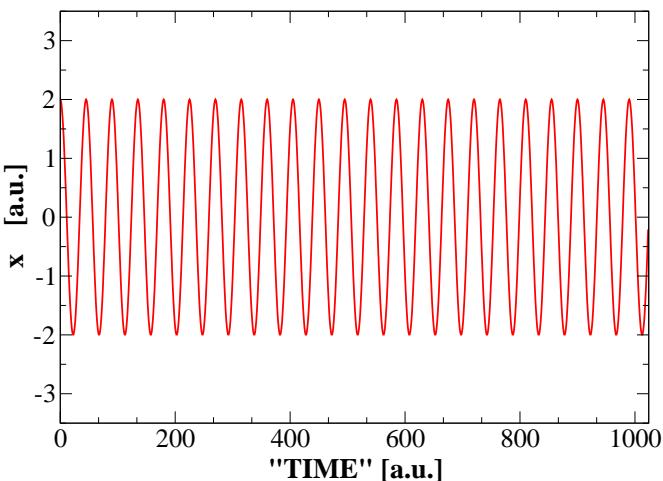
Disadvantage:

- we can reconstruct the particle trajectory only at the BPM locations \Rightarrow Poincaré section.

Physical introduction

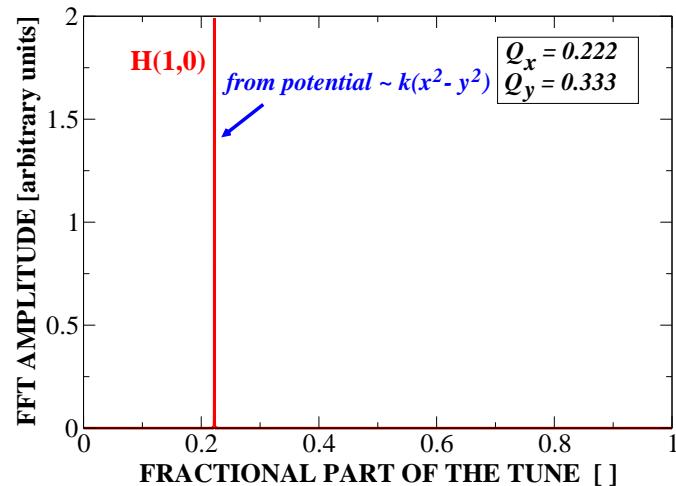
Particle oscillation (after kicking) under the normal (focusing) quadrupole field $V = \frac{k_1(s)}{2}(x^2 - y^2)$

HORIZONTAL PARTICLE OSCILLATIONS



Trajectory

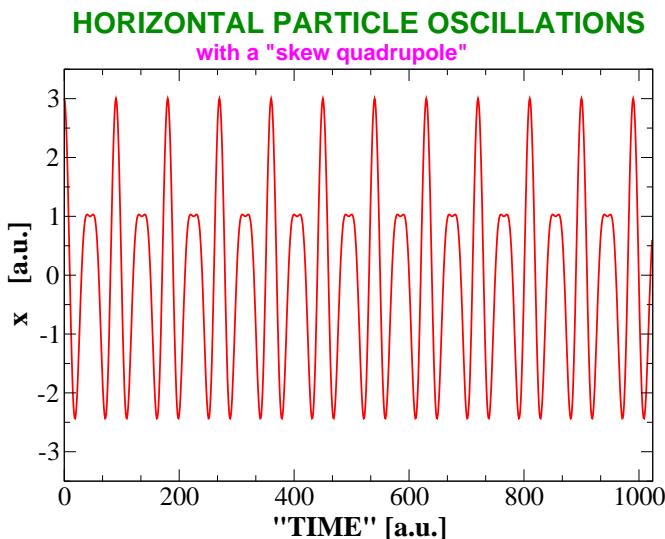
HORIZONTAL SPECTRUM OF $h_x = x + ip_x$



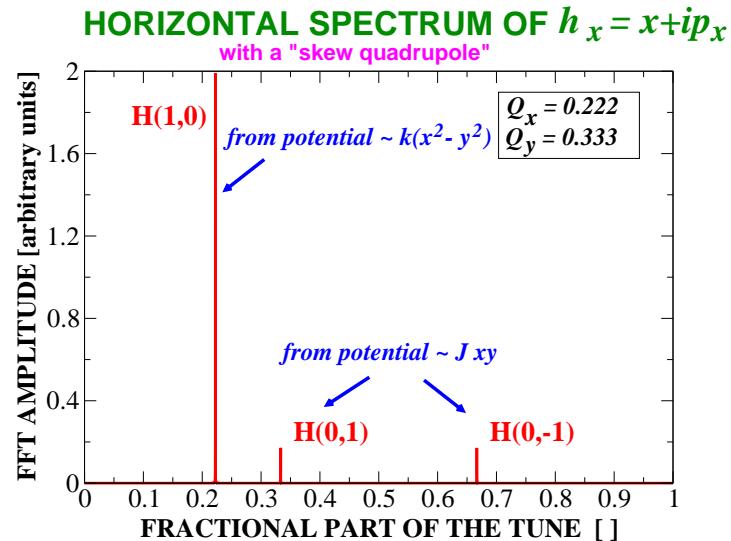
Spectrum

Physical introduction

Particle oscillation (after kicking) under the normal (focusing) and skew quadrupole field $V = \frac{k_1(s)}{2}(x^2 - y^2) - J_1 xy\delta(s - s_0)$



Trajectory



Spectrum

Physical introduction

How are the secondary spectral lines related to the integrated gradients $K_n \quad J_n$?

- assumption: pencil beam \Rightarrow single particle dynamics
- from two BPMs we get position and momentum of the beam centroid turn-by-turn:

$$x, \quad p_x$$

- which are transformed in the Courant-Snyder coordinates:

$$\tilde{x}, \quad \tilde{p}_x$$

- We look at the spectrum of the complex (experimental) variable

$$h_{x,+} = \tilde{x} + i\tilde{p}_x = Ae^{i2\pi NQ_x} + \sum_{n=1}^{\infty} a_n e^{i2\pi(b_n Q_x + c_n Q_y)}$$

- $h_{x,+}, h_{x,-}, h_{y,+}, h_{y,-}$ Complex Courant-Snyder coordinates

Physical introduction

- The amplitude of secondary lines a_n are related to the multipole gradients via Normal Form

$$a_n \Rightarrow a_{jklm}, \quad n = j + k + l + m$$

$$a_{jklm} \propto f_{jklm} \quad f_{jklm} \propto h_{jklm}$$

$$f_{jklm} = \frac{h_{jklm}}{1 - e^{2i\pi[(j-k)Q_x + (l-m)Q_y]}} \quad \text{Resonance Driving Terms}$$

$$\text{skew quadrupoles:} \quad h_{1001} = \frac{J_1(\beta_x \beta_y)^{1/2}}{4}$$

$$\text{normal sextupoles:} \quad h_{3000} = \frac{K_2(\beta_x)^{3/2}}{48}$$

Physical introduction

Useful relation to benchmark the code, when a single skew quadrupole is switched on:

$$\frac{|f_{1010}|}{|f_{1001}|} = \frac{|\sin(\pi[-Q_x + Q_y])|}{|\sin(\pi[-Q_x + Q_y])|}$$

What happens when many multipoles are present in the ring?

The spectral lines (and the corresponding Driving Terms) are subjected to a jump when a multipole is present:

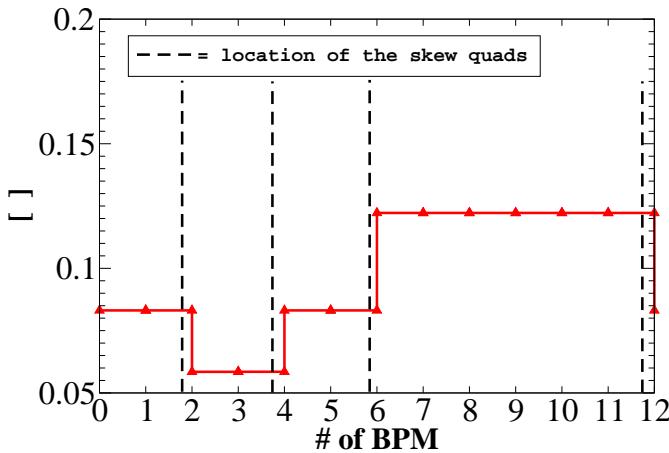
SIS-18 benchmarks with four skew quads: [CLICK HERE](#)

Physical introduction

The Hamiltonian terms are inferred by (Rogelio's formula):

$$h_{\alpha,jklm} = f_{jklm}^{(\alpha)} e^{i[(j-k)(\phi_{x,\alpha-1} - \phi_{x,\alpha}) + (l-m)(\phi_{y,\alpha-1} - \phi_{y,\alpha})]} - f_{jklm}^{(\alpha-1)}$$

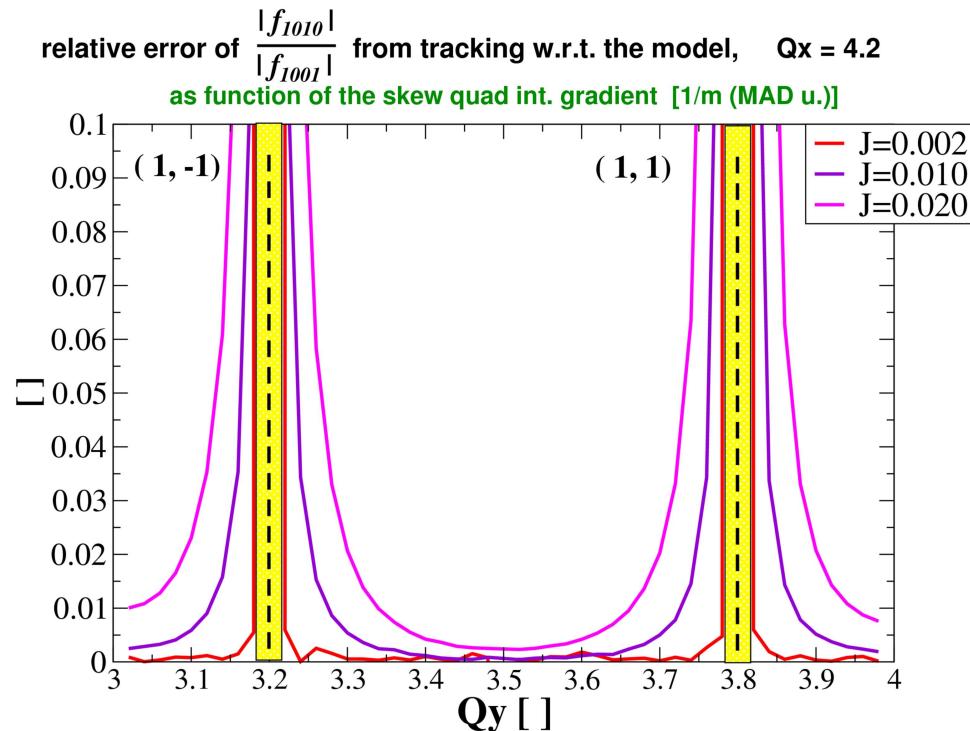
amplitude of f_{1001} Vs long. position



$$h_{\alpha,1001} = \frac{J_{\alpha,1}(\beta_{\alpha,x}\beta_{\alpha,y})^{1/2}}{4}$$

Limits of the model

- perturbation theory \Rightarrow small magnet strengths (K_n, J_n);
- non-resonant theory \Rightarrow tunes far from resonance conditions;
- no more than one multipole between two BPMs**;



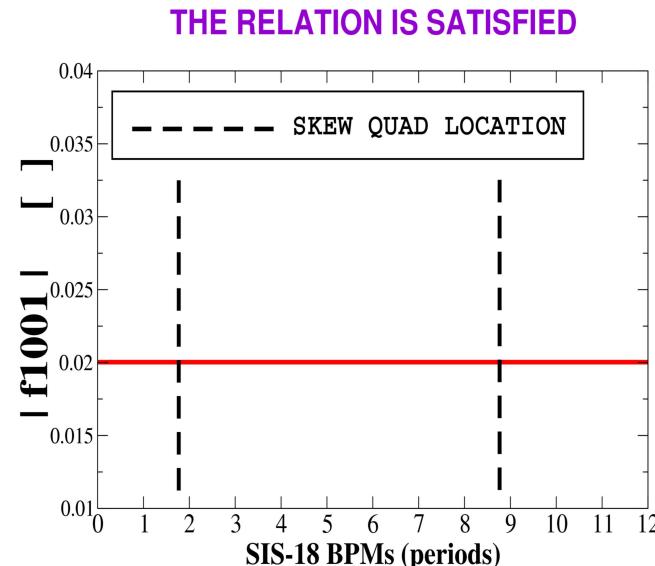
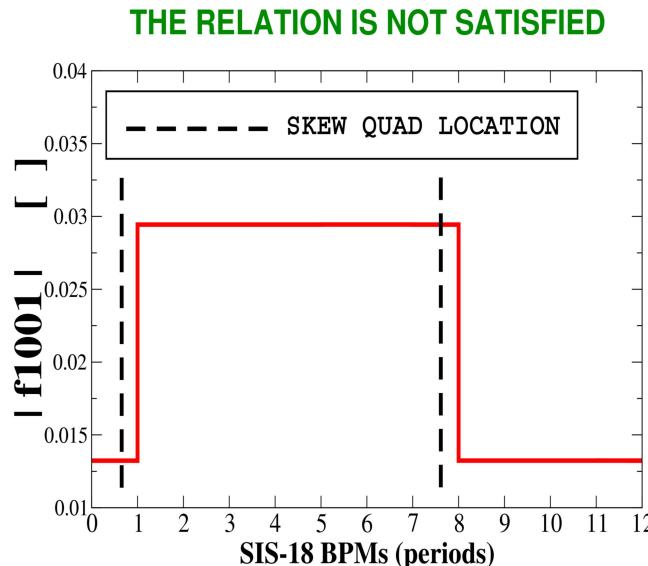
Limits of the model

The Shadow Effect!

If two skew quads are switched on and their phase advances ϕ_x , ϕ_y satisfy

$$(\phi_{1x} - \phi_{1,y}) - (\phi_{2,x} - \phi_{2,y}) = \pi(Q_x - Q_y)$$

no jump is observable (similar relations for higher multipoles)



Code description

- built using the MIMAC libraries (optics and tracking);
- reads the MAD-like input file;
- calculates the matched optical functions;
- tracks a single particle along the ring;
- optionally reads output file from GSI control room or MAD;
- the FFT (Hanning filter, Bazzani et al.) is performed;
- the Driving and the Hamiltonian terms are inferred;
- the magnet strengths are calculated.

Experimental issues

- Linear machine must correspond to the model (optical functions, tunes);
- “clean” spectrum (everything is based on the line amplitudes);
- chromaticity and amplitude detuning should be corrected;
- pencil beam (i.e. cooled and “small”);
- BPM acquisition turn-by-turn for at least 1024 turns;
- low BPM noise (the Hanning filter is ineffective when noise occurs);

The SIS-18 at GSI

- 217 m synchrotron of 12 periods, $E_{inj} = 11.4 \text{ MeV/u}$;
- $Q_x = 4.29$ $Q_y = 3.28$, below transition;
- 12 BPMs at the end of each period ($\beta_x = 12.9$, $\beta_y = 13.6$);
- 8 independent skew quadrupoles (1,2,4,6,7,8,10,12);
- 12 (6+6) independent normal sextupoles for chromaticity correction and extraction (1,3,5,7,9,11);
- BPM resolution at 250 μA 0.5 mm;
- typical operation: $^{238}\text{U}^{+73,28}$, $\approx 10^8$ particles, $\sigma_p \approx 10^{-3}$ (without cooling), $\sigma_z \approx 20 \text{ m}$.

SIS-18 benchmark: single skew quadrupole

$$|f_{1001}/f_{1010}|$$

MODEL	TRACKING	REL. ERROR (ABS)
$0.63923 \cdot 10^{-1}$	$0.64340 \cdot 10^{-1}$	$0.65262 \cdot 10^{-2}$

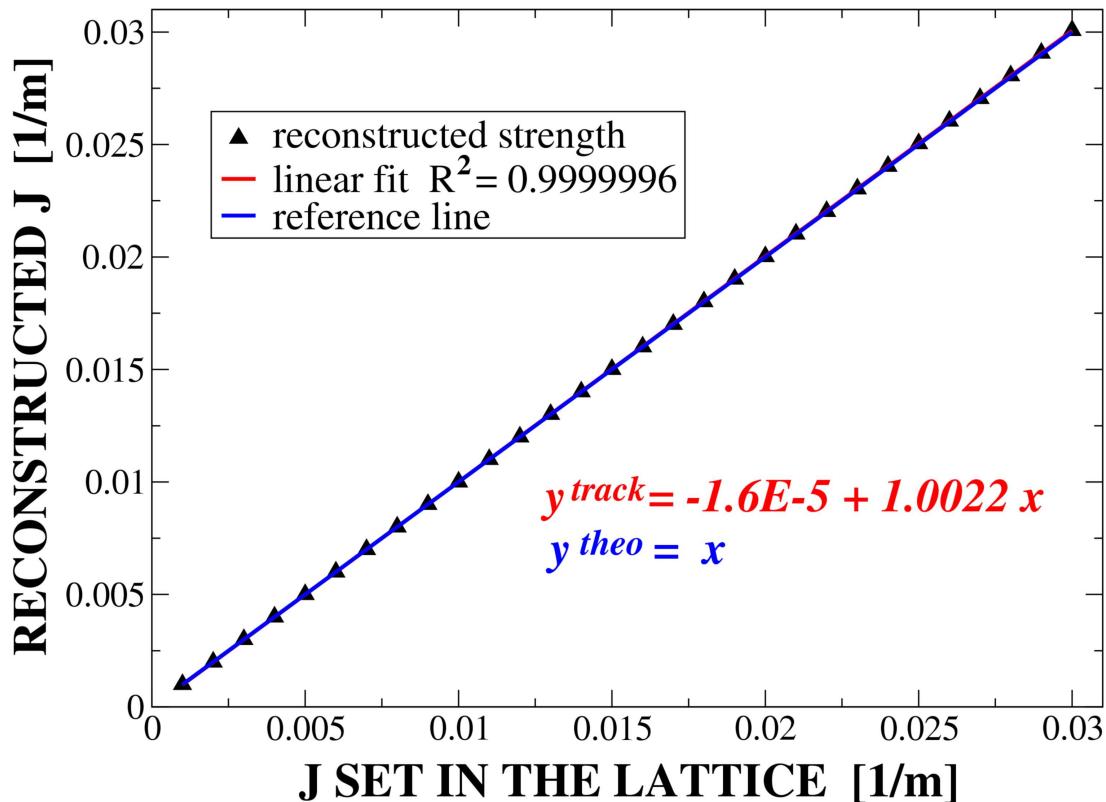
SKEW QUAD GRADIENT RECONSTRUCTION [m⁻¹]

J_1 SET IN THE LATTICE	J_1 RECONSTRUCTED
$-.20000 \cdot 10^{-2}$	$-.19947 \cdot 10^{-2}$

$$Q_x = 4.27, Q_y = 3.29$$

SIS-18 benchmark: single skew quadrupole

With several strengths set in the lattice



SIS-18 benchmark: all the skew quadrupoles

SKEW QUAD GRADIENTS RECONSTRUCTION [m⁻¹]

PERIOD	J_1 SET IN THE LATTICE	J_1 RECONSTRUCTED
--------	--------------------------	---------------------

1	$5.000 \cdot 10^{-3}$	$4.952 \cdot 10^{-3}$
2	$5.000 \cdot 10^{-3}$	$5.014 \cdot 10^{-3}$
4	$5.000 \cdot 10^{-3}$	$4.980 \cdot 10^{-3}$
6	$5.000 \cdot 10^{-3}$	$5.004 \cdot 10^{-3}$
7	$5.000 \cdot 10^{-3}$	$5.019 \cdot 10^{-3}$
8	$5.000 \cdot 10^{-3}$	$4.981 \cdot 10^{-3}$
10	$5.000 \cdot 10^{-3}$	$5.001 \cdot 10^{-3}$
12	$-2.000 \cdot 10^{-4}$	$-2.006 \cdot 10^{-4}$

$$Q_x = 4.20, Q_y = 3.40$$

SIS-18 benchmark: all the skew quadrupoles

SKEW QUAD GRADIENTS RECONSTRUCTION [m⁻¹]

PERIOD	J_1 SET IN THE LATTICE	J_1 RECONSTRUCTED
--------	--------------------------	---------------------

1	$5.000 \cdot 10^{-3}$	$4.855 \cdot 10^{-3}$
2	$5.000 \cdot 10^{-3}$	$5.090 \cdot 10^{-3}$
4	$5.000 \cdot 10^{-3}$	$5.053 \cdot 10^{-3}$
6	$5.000 \cdot 10^{-3}$	$4.894 \cdot 10^{-3}$
7	$5.000 \cdot 10^{-3}$	$4.938 \cdot 10^{-3}$
8	$5.000 \cdot 10^{-3}$	$5.053 \cdot 10^{-3}$
10	$5.000 \cdot 10^{-3}$	$5.105 \cdot 10^{-3}$
12	$-2.000 \cdot 10^{-4}$	$-1.920 \cdot 10^{-4}$

$$Q_x = 4.255, Q_y = 3.29 !!$$

SIS-18 benchmark: several normal sextupoles

NORMAL SEXTUPOLE GRADIENTS RECONSTRUCTION [m⁻²]

$$Q_x = 4.28, Q_y = 3.29 !!$$

PERIOD	K_2 SET IN THE LATTICE	K_2 RECONSTRUCTED
--------	--------------------------	---------------------

3	$-2.000 \cdot 10^{-1}$	$-2.004 \cdot 10^{-1}$
5	$-2.000 \cdot 10^{-1}$	$-2.002 \cdot 10^{-1}$
7	$-2.000 \cdot 10^{-1}$	$-1.999 \cdot 10^{-1}$
9	$-2.000 \cdot 10^{-1}$	$-1.997 \cdot 10^{-1}$
11	$1.000 \cdot 10^{-1}$	$9.973 \cdot 10^{-2}$

The RHIC skew quadrupole families

- SQ**C2B: 12 skew quads used as corrector in the triplets between Q2 and Q3, one per period, independently powered;
- SQSK**: 6 groups of skew quads, used to minimize ΔQ_{min} , several per period, shared power supplies ($J(12) = -J(6)$, $J(2) = -J(8)$, $J(4) = -J(10)$);
- SQSK**LC: 12 skew quads, one per period, used in the past for skew quad correction studies, independently powered;
- SQSK: unpowered skew quad family.

Information from Steven Tepikian.

RHIC benchmark: single skew quadrupole

$$|f_{1001}/f_{1010}|$$

MODEL	TRACKING	REL. ERROR (ABS)
0.61803	0.62181	$0.61078 \cdot 10^{-2}$

SKEW QUAD GRADIENT RECONSTRUCTION [m⁻¹]

J_1 SET IN THE LATTICE	J_1 RECONSTRUCTED
$0.10000 \cdot 10^{-1}$ [m ⁻¹]	$0.10014 \cdot 10^{-1}$ [m ⁻¹]

$$Q_x = 28.20, Q_y = 29.40$$

RHIC benchmark: single skew quadrupole

$$|f_{1001}/f_{1010}|$$

MODEL	TRACKING	REL. ERROR (ABS)
$0.63923 \cdot 10^{-1}$	∞	∞

SKEW QUAD GRADIENT RECONSTRUCTION [m⁻¹]

J_1 SET IN THE LATTICE	J_1 RECONSTRUCTED
$0.10000 \cdot 10^{-1}$ [m ⁻¹]	$-0.10517 \cdot 10^8$ [m ⁻¹]

$Q_x = 28.22$, $Q_y = 29.23$!!!

RHIC benchmark: several skew quadrupoles

GRADIENTS RECONSTRUCTION: SQ**LC FAMILY [m⁻¹]

PERIOD	J_1 SET IN THE LATTICE	J_1 RECONSTRUCTED
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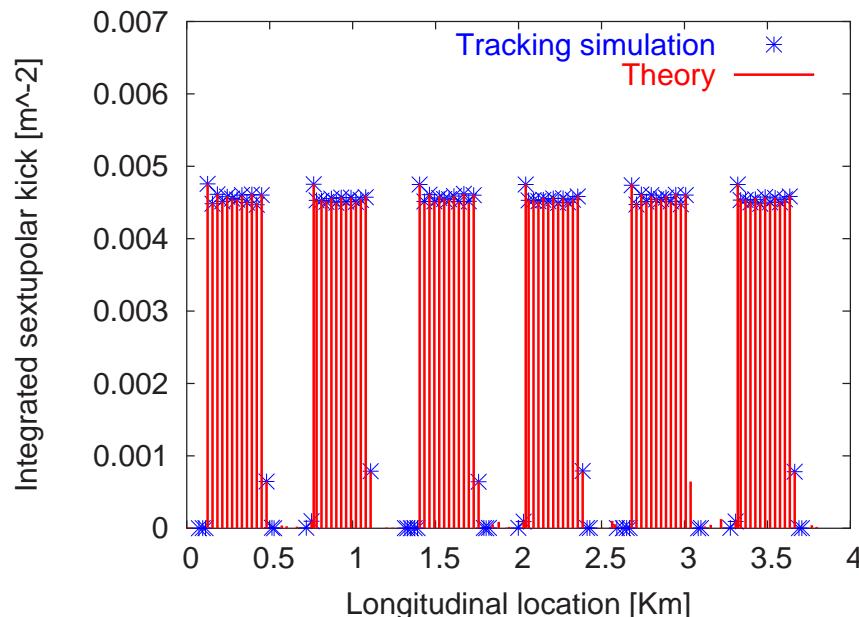
6	$1.000 \cdot 10^{-2}$	$1.001 \cdot 10^{-2}$
7	$1.000 \cdot 10^{-2}$	$9.967 \cdot 10^{-3}$
8	$1.000 \cdot 10^{-2}$	$9.996 \cdot 10^{-3}$
9	$1.000 \cdot 10^{-2}$	$9.826 \cdot 10^{-3}$
10	$1.000 \cdot 10^{-2}$	$9.828 \cdot 10^{-3}$
11	$1.000 \cdot 10^{-2}$	$9.703 \cdot 10^{-3}$
12	$1.000 \cdot 10^{-2}$	$9.714 \cdot 10^{-3}$
1	$1.000 \cdot 10^{-2}$	$9.960 \cdot 10^{-3}$
2	$1.000 \cdot 10^{-2}$	$9.838 \cdot 10^{-3}$
3	$1.000 \cdot 10^{-2}$	$9.950 \cdot 10^{-3}$
4	$1.000 \cdot 10^{-2}$	$1.005 \cdot 10^{-2}$
5	$5.000 \cdot 10^{-3}$	$4.988 \cdot 10^{-2}$

$$Q_x = 28.20, Q_y = 29.25$$

RHIC benchmark: normal sextupoles

When N sextupoles are present between two BPMs only the integrated strength can be inferred:

$$\sum_{q=1}^N e^{-2i\phi_{xq}} \sin \phi_{xq} h_{q,3000} \quad h_{q,3000} = \frac{K_{q,2}(\beta_{q,x})^{3/2}}{48}$$



CONCLUSIONS

- a beam-based method to infer the magnet strength has been proposed;
- the method is based on the turn-by-turn BPM acquisition and the Resonance Driving terms measurement;
- numerical benchmarks confirm its reliability;
- from the experimental point of view “perfect” knowledge of the linear machine, “high” BPM resolution, chromaticity and amplitude detuning correction are required;
- the method is based on a non-resonant model \Rightarrow it is sensitive to the working point (tunes).

optical functions

```
#----- SIS-skew quads -----
# EL # beta_x[m]    alpha_x[]    nu_x[]      beta_y[m]   alpha_y[]   nu_y[]
#-----
```

1	10.24	-1.025	0.270	8.797	-0.358	0.197
2	11.44	-2.513	0.698	20.61	5.945	0.534
3	11.44	-2.513	1.410	20.61	5.945	1.083
4	11.44	-2.513	2.124	20.61	5.945	1.631
5	11.77	-1.166	2.420	9.362	-0.448	1.854
6	11.44	-2.513	2.837	20.61	5.945	2.179
7	11.44	-2.513	3.551	20.61	5.945	2.728
8	11.44	-2.513	4.264	20.61	5.945	3.276


```
#----- SIS-BPM -----
# beta_x[m]    beta_y[m]    delta_phi_x  delta_phi_y
#-----
```

12.7223516	13.4108936	2.24100046	1.72263919
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```

#----- RHIC-skew quads SQSK**LC -----
# EL # beta_x[m] alpha_x[] nu_x[] beta_y[m] alpha_y[] nu_y[]
#-----
 178  33.45    1.504   1.377   11.71   -0.698   0.95
 518  34.31    1.701   3.825   11.58   -0.545   3.55
 870  11.75    0.587   6.073   36.95   -1.627   5.79
1188  11.43    0.567   8.528   37.99   -1.979   8.43
1556  35.11    1.568  10.802   11.20   -0.619  10.71
1896  35.44    1.719  13.252   10.92   -0.542  13.31
2214  12.70    0.678  15.500   36.88   -1.612  15.56
2532  12.49    0.615  17.959   37.56   -1.959  18.19
2891  32.26    1.417  20.236   11.31   -0.618  20.47
3231  32.65    1.616  22.681   10.87   -0.529  23.08
3584  12.35    0.619  24.928   39.41   -1.719  25.32
3902  11.94    0.567  27.385   37.33   -1.881  27.95

```